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The Sphaleron Rate in the "Symmetric" Electroweak Phase

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Abstract

I calculate the rate of $B + L$ violating processes in the "symmetric" phase of the Standard Model by means of semiclassical methods, which are based on an appropriately resummed loop expansion within the dimensionally reduced theory. The rate is found to reach its asymptotic form $\Gamma/V = c(\alpha_W T)^4$ at a temperature about three times the critical temperature of the electroweak phase transition, with a coefficient $c \sim 0.01$ at the one-loop level. The order of magnitude of c is sensitive to higher order corrections to the dynamically generated vector boson mass if these are greater than 20%. The temperature below which baryon number dissipation is in thermal equilibrium in the early universe is estimated to be $T_* \sim 10^{11}$GeV.

The origin of the observed baryon asymmetry of the universe still remains a puzzle, despite many attempts to explain it [1]. Whatever be the mechanism for the generation of the baryon asymmetry realized by nature, its final value was determined by the $B+L$ violating processes within the Standard Model which are active at the epoch of the electroweak phase transition [2]. A particularly important quantity is the rate at which $B+L$ violation proceeds in the "symmetric" phase of the Standard Model. This quantity determines, together with the set of conserved charges, how much of any primordial $B+L$ and/or B excess survives until the universe undergoes the electroweak phase transition. Furthermore, it also enters all scenarios of electroweak baryogenesis [1].

As a starting point for discussions of B+L violation at high temperatures it is customary to approximate the electroweak theory by an SU(2) Higgs model. In the broken phase this theory has a saddle point solution of the static classical field equations, the sphaleron [3], representing the top of a potential barrier between topologically different vacuum sectors. Its energy is given by

$$E_S = B(\lambda/g^2)\frac{2m(T)}{\alpha_W} \ , \tag{1}$$

where $B(\lambda/g^2)$ is a slowly varying function of the ratio of the coupling constants and $m(T)$ denotes the temperature dependent vector boson mass. The rate of B+L violating processes is related to the rate at which the thermally excited gauge Higgs system crosses the barrier, which may be calculated semiclassically [4]-[8] by means of the Langer-Affleck theory [9] according to

$$\Gamma = \frac{\mid \omega_- \mid}{\pi T}\ \mathrm{Im}F \approx \frac{\mid \omega_- \mid}{\pi}\frac{\mathrm{Im}Z_S}{Z_0} \ . \tag{2}$$

Here the free energy F of the system is evaluated by expanding in Gaussian fluctuations around the sphaleron and the vacuum solutions, respectively, and ω_-^2 denotes the negative eigenvalue of the unstable mode. In the symmetric phase, due to the vanishing of the vacuum expectation value and the vector boson mass, no non-trivial solution of the classical field equations is known, the loop expansion breaks down and a semiclassical calculation seems impossible. The general form of the rate may nevertheless be inferred from dimensional analysis to be [4, 5]

$$\frac{\Gamma}{V} = c\ (\alpha_W T)^4 \ . \tag{3}$$

The dimensionless coefficient c was found in lattice simulations to be of order $\mathcal{O}(0.1) - \mathcal{O}(1)$ [10][1]. However, these simulations have been criticised recently [12] and it is not

[1] For a more detailed review of the situation in the symmetric phase see [11]

clear how reliable this result for c is. It is therefore particularly desirable to have an independent analytical calculation of the rate in the symmetric phase.

A semiclassical calculation for temperatures above the critical temperature T_c of the electroweak phase transition becomes possible if a transverse mass $m = m_0 g^2 T$ is generated dynamically for the spatial components of the vector field. Such a "magnetic mass" screens infrared singularities of the order $\sim g^2 T$, thus making a loop expansion possible. However, the coefficient m_0 of this mass is entirely non-perturbative [13]. With a massive vector boson, one expects again a potential barrier between the different winding number sectors of the theory, peaked at a saddle point providing the means to use semiclassical techniques. On the other hand, the presence of a barrier leads to a Boltzmann factor $\exp(-\beta E_S)$ at inverse temperatures $\beta = 1/T$ suppressing the rate of transitions [2]. With a temperature dependent mass $m \sim g^2 T$, the sphaleron energy is of the form

$$E_S = 8\pi m_0 B(\lambda/g^2) T \ . \tag{4}$$

Consequently, the Boltzmann factor is temperature independent and the strength of the supression depends exponentially on the coefficients of the vector boson mass, m_0, and the sphaleron energy, B. Based on this observation a non-linear sigma model describing massive vector bosons was studied in Ref. [14], and the rate of sphaleron transitions was estimated as a function of the coefficient of the magnetic mass. The main motivation for this model was the fact that it implements the vector boson mass in a gauge invariant manner, while the Higgs degrees of freedom with their thermal masses of the order $\sim gT$ may be viewed as decoupling heavy particles compared to the scale of interest $\sim g^2 T$. The maximal rate achievable in this model was found to be $c \sim \mathcal{O}(0.01)$ for $m_0 = 0.1$, but smaller by many orders of magnitude for other values of m_0.

Non-perturbative effects in finite temperature field theory are related to the infrared behaviour of the corresponding three-dimensional theory. In a recent paper [15] dynamical mass generation was studied in a gauge invariant manner in the three-dimensional SU(2) Higgs model. Supplementing mass resummations with vertex resummations led to a set of gauge independent gap equations for the Higgs boson and vector boson masses. For parameter values generally associated with the "symmetric" phase, the gap equations exhibit solutions with non-vanishing vacuum expectation value and vector boson mass. This suggests that the "symmetric" phase is again a Higgs phase, just with modified parameters. In subsequent work [16] a comparison of the gap equation approach as applied to the abelian and non-abelian Higgs model established that the appearance of a non-vanishing vacuum expectation value and vector boson mass in the "symmetric"

phase is specific to the non-abelian model and hence not an artifact of the resummation scheme.

In this letter I shall assume that the picture developed in Ref. [15] gives a reliable description of the "symmetric" phase and explore its consequences for the sphaleron rate. Given the structure of the resummed action, the calculation of the rate in the modified loop expansion is straightforward. Since the resummed three-dimensional theory appears again to be in a Higgs phase, all that needs to be done is to match its parameters properly to previous calculations of the rate in the broken phase [6]-[8]. In this paper I work with the effective three-dimensional theory without fermions, following the calculation in Ref. [6]. This is particularly convenient because in this case the results of [15] can be used directly. As will be discussed after the presentation of this limiting case, a complete treatment including fermions and keeping the non-zero Matsubara modes, analogous to that in Ref. [8], should also be possible in principle.

In order to illustrate the reduction of the rate calculation in the "symmetric" phase to the known ones in the broken phase it is necessary to briefly outline the approach of Ref. [15]. It is well known that for high temperatures and small enough couplings the SU(2) Higgs model may be perturbatively reduced to yield an effective three-dimensional model

$$S_3 = \int d^3x \left[\frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\Phi)^\dagger D_i\Phi + \mu_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2\right] , \tag{5}$$

where the effective parameters are related to the original four-dimensional ones by [17, 18]

$$\begin{aligned} g_3^2 &= g^2(T)T \ , \ \lambda_3 = \left(\lambda(T) - \frac{3}{128\pi}\sqrt{\frac{6}{5}}g^3(T) + \mathcal{O}(g^4,\lambda^2)\right) T \ , \\ \mu_3^2 &= \left(\frac{3}{16}g^2(T) + \frac{1}{2}\lambda(T) - \frac{3}{16\pi}\sqrt{\frac{5}{6}}g^3(T) + \mathcal{O}(g^4,\lambda^2)\right)(T^2 - T_b^2) \, . \end{aligned} \tag{6}$$

Here T_b denotes the "barrier temperature",

$$T_b = \left[\frac{16\lambda v^2}{3g^2(T) + 8\lambda(T)}\right]^{1/2} , \tag{7}$$

which coincides with the critical temperature T_c for a second-order phase transition but is slightly lower than T_c for a first-order phase transition [19]. Since the main interest here is in temperatures well above the phase transition, this difference is immaterial in what follows. At tree level, for $\mu_3^2 < 0$, the Higgs field in eq. (5) develops a vacuum expectation value $v_3^2 = -\mu_3^2/\lambda_3$, the system is in the Higgs phase, has a sphaleron solution and the loop expansion in g_3^2/m with $m = g_3 v_3/2$ is convergent. For $\mu_3^2 > 0$, $v_3 = 0$ and a loop

expansion in terms of the parameter g_3^2/m is not possible. This situation can be remedied by rearranging the perturbation series,

$$S_3 = S_3 + \delta S_3 - \delta S_3 \equiv S_{3R} - \delta S_3 \,, \tag{8}$$

where δS_3 contains mass and vertex corrections from higher orders in the ordinary loop expansion. In the modified expansion one-loop calculations are performed starting from S_{3R} while the terms included in $-\delta S_3$ are treated as counterterms which have to be taken into account in higher orders. Calculating self-energies in this scheme leads to a coupled set of non-linear gap equations in which the physical pole masses of the vector boson (m) and the Higgs boson (M) as well as the the vacuum expectation value of the Higgs field are determined self-consistently as functions of the parameters of the theory,

$$\begin{aligned} m^2 &= f(m/M, v_3;\ \lambda_3/g_3^2, \mu_3^2/g_3^4) \\ M^2 &= g(m/M, v_3;\ \lambda_3/g_3^2, \mu_3^2/g_3^4) \\ v_3 &= h(m/M, v_3;\ \lambda_3/g_3^2, \mu_3^2/g_3^4) \,, \end{aligned} \tag{9}$$

where the explicit forms of the functions f, g, h are given in Ref. [15]. The resummed action may be written in terms of the solutions of these equations as

$$S_{3R} = \int d^3x \left[\frac{1}{4} F_{ij}^a F_{ij}^a + (D_i\Phi)^\dagger D_i\Phi + \mu_{3R}^2 \Phi^\dagger\Phi + \lambda_{3R}(\Phi^\dagger\Phi)^2 \right] , \tag{10}$$

with the resummed parameters

$$\lambda_{3R} = \frac{g^2 M^2}{8m^2}, \quad \mu_{3R}^2 = -\frac{1}{2} M^2 \,. \tag{11}$$

From this form of the action it follows that the "symmetric" phase is again a Higgs phase, since M^2 is a physical mass and always positive. The first-order phase transition with a jump in the masses found for small values of λ_3/g_3^2 changes to a smooth crossover at large λ_3/g_3^2. This reorganisation of the perturbative series assumes that the classical vacuum solution of the field equations is not the true ground state of the theory, as is also indicated by the breakdown of the loop expansion. Instead, it perturbs around the true vacuum containing quantum interactions. Correspondingly, the classical field equations have no sphaleron solution for $\mu_3^2 > 0$, while the field equations derived from eq. (10) with $\mu_{3R}^2 < 0$ do have a non-trivial solution. Clearly the form of these field equations and their sphaleron solution is the same as that of the classical ones in the broken phase, with the replaced parameters $\lambda_3 \to \lambda_{3R}$ and $\mu_3^2 \to \mu_{3R}^2$. In particular this means that the calculation of the rate now is the same as that in the broken phase.

In order to transform the action (10) to the form used in Ref. [6] coordinates and fields have to be rescaled to be dimensionless,

$$x_i \to \xi_i/2m\,, \quad W_i \to 2mW_i\,, \quad \Phi \to 2m\Phi\,. \tag{12}$$

Using $m = g_3 v_{3R}/2$ with $v_{3R}^2 = -\mu_{3R}^2/\lambda_{3R}$ then leads to

$$S_{3R} = \frac{2m^2}{g_3^2}\int d^3\xi\,\left[\frac{1}{4}F_{ij}^a F_{ij}^a + (D_i\Phi)^\dagger D_i\Phi + \frac{\lambda_{3R}}{g_3^2}\left(\Phi^\dagger\Phi - \frac{1}{2}\right)^2\right]\,. \tag{13}$$

This action is now expanded in small fluctuations around a background field,

$$W_i = W_i^b + w_i\,, \quad \Phi = \Phi^b + \eta\,, \tag{14}$$

where the upper index b denotes the vacuum or sphaleron solution and w_i, η are the fluctuation fields. To eliminate gauge degrees of freedom, a gauge has to be fixed. This is achieved by imposing the $R_{\xi=1}$ background gauge condition

$$D_i(W^b)w_i + \mathrm{i}(\Phi^\dagger\tau\eta - \eta^\dagger\tau\Phi) = 0\,, \tag{15}$$

with the Fadeev-Popov determinant

$$\Delta_{FP} = \det\left[-D_i^2 + \frac{1}{2}\Phi^\dagger\Phi\right]\,, \tag{16}$$

and adding the corresponding gauge fixing and Fadeev-Popov terms to the action,

$$S_{eff} = S_{3R} + S_{gf} + S_{FP}\,. \tag{17}$$

In the Gaussian approximation only terms quadratic in the fields are retained and functional integration over the fields takes eq. (2) to

$$\frac{\Gamma}{V} = \frac{|\,\omega_-\,|}{2\pi}\frac{8\pi^2 N_{tr}^3 N_{rot}^3}{(g^2\beta)^3}\mathrm{e}^{-\beta E_S}(2m\beta)^6\kappa\,, \tag{18}$$

where κ contains the fluctuation determinants

$$\kappa = \mathrm{Im}\left(\frac{\det(\delta^2 S_{eff}/\delta\phi^2)\mid_{\phi=\phi_0}}{\det'(\delta^2 S_{eff}/\delta\phi^2)\mid_{\phi=\phi_S}}\right)^{1/2}\left(\frac{\Delta_{FP}\mid_{\phi=\phi_S}}{\Delta_{FP}\mid_{\phi=\phi_0}}\right)\,. \tag{19}$$

In this expression ϕ generically stands for the scalar and vector fields of the theory, while the indices $S, 0$ label sphaleron and vacuum configurations, respectively. The prime on the determinant indicates that the sphaleron translational and rotational zero modes are to be omitted from its evaluation. These modes have to be integrated separately by the method of collective coordinates leading to the factor $8\pi^2 N_{tr}^3 N_{rot}^3/(g^2\beta)^3$ in eq. (18),

where N_{tr}^3 and N_{rot}^3 are normalization integrals of the zero modes depending on λ_3/g_3^2 (for a more detailed explanation of the various factors see [6] and references therein). Note that in this formula the ratio of Fadeev-Popov determinants enters with a power of one as opposed to the calculation in [6], where it enters with a power of 1/2. The reason is that in Ref. [6] there are still W_0's in the theory. Gaussian integration over those fields produces a factor $\Delta_{FP}^{-1/2}$. In the current approach the W_0's have been integrated out already in the construction of the effective three-dimensional theory (5) and their contributions are included in its parameters.

The evaluation of eq. (18) is now straightforward. The normalization factor $N_{tr}^3 N_{rot}^3$, the negative mode ω_-^2, the coefficient of the sphaleron energy B and the fluctuation determinants κ are all given in Ref. [6] as functions of λ_3/g_3^2. In the resummed scheme employed here, they are functions of λ_{3R}/g_3^2 instead, depending on the original parameters of interest through the resummed ones according to eqs. (11), (9) and (6). For any given set of parameters $\{\lambda/g^2, T/T_b\}$ the matching equations (6) give the corresponding three-dimensional parameters $\{\lambda_3/g_3^2, \mu_3^2/g_3^4\}$. These determine the solutions of the gap equations which then lead to the required resummed parameters $\{\lambda_{3R}/g_3^2, \mu_{3R}^2/g_3^4\}$ of the action (10). The result of this procedure is displayed in Fig. 1, where the temperature dependence of λ_{3R}/g_3^2 is shown for $\lambda/g^2 = 1/8$. Here and in the following the slowly running gauge coupling was taken to be a constant $g = 0.67$ for simplicity. The coefficient of the vector boson mass assumes the constant value $m_0 = 0.28$ for $T > 1.5T_b$, where it is also rather insensitive to variations of λ/g^2 [15]. For any value of λ_{3R}/g_3^2 one may finally read off all the quantities entering eq. (18) from the graphs given in Ref. [6]. There are so far three complete calculations of the fluctuation determinant κ. The first of these [6] seems to disagree with the other ones [7, 8] for small values of λ_3/g_3^2. However, the results of Ref. [7] and Ref. [8] agree within 10% accuracy. I have therefore chosen to take the data for the contributions to κ from Ref. [8][2].

Denoting $\mid \omega_- \mid /m$ by $\mid \bar{\omega}_- \mid$, the rate (18) may be rewritten in the form of eq. (3) with

$$c = \mid \bar{\omega}_- \mid 2^6 (4\pi)^5 N_{tr}^3 N_{rot}^3\, m_0^7\, \kappa\, \exp[-8\pi m_0 B]\ . \tag{20}$$

Note that, through its dependence on λ_{3R}/g_3^2 and the matching equations, c is temperature dependent in general. Fig. 2 shows the result of a numerical interpolation between several data points obtained in the manner described above. The solid line is the result for $\lambda/g^2 = 1/8$ or a zero temperature tree-level mass ratio of $m_H^2/m_W^2 = 1$, and the dashed curve is for $\lambda/g^2 = 1/64$ or $m_H^2/m_W^2 = 1/8$. Close to the phase transition c is strongly

[2] I am grateful to K. Goeke and P. Sieber for generously providing me with their data

temperature dependent, dropping exponentially as T_b is approached. For temperatures higher than $\sim 3T_b$ the sphaleron rate reaches its asymptotic form (3) with constant c, where it is also rather insensitive to the value of the Higgs mass. In this asymptotic region the constant coefficient takes on the value $c \sim 0.01$. It should be emphasized that the onset of the asymptotic behaviour is a non-trivial prediction within the modified loop expansion, relying on the temperature dependence of the resummed scalar coupling as shown in Fig. 2.

Comparing with the results obtained earlier for the non-linear sigma model [11] one finds that keeping the Higgs degree of freedom enhances the rate considerably, even though it apparently has little influence on the dynamics and the mass of the vector field [15]. The reason is that the relevant saddle point solution of the non-linear sigma model has, at the same value of m_0, substantially higher energy than the sphaleron solution of the Higgs model, resulting in a stronger Boltzmann suppression. Therefore the Higgs degree of freedom may not be neglected in a semiclassical treatment, even though it seems to play only a marginal role dynamically.

In the case of $B-L=0$, the actual baryon number dissipation rate is related to the sphaleron rate by [4]

$$\Gamma_B = \frac{dN_B}{N_B dt} \approx -\frac{13}{2} n_f \beta^3 \frac{\Gamma}{V} \ , \tag{21}$$

for n_f fermion generations. The temperature T_* at which the sphaleron processes drop out of equilibrium may be estimated by equating the dissipation rate Γ_B with the Hubble expansion rate of the universe. Well above the phase transition the latter is $H \approx 16.6/(\beta^2 M_{Pl})$ [20] and one obtains

$$T_* \approx \frac{1}{16.6} \frac{13}{2} n_f M_{Pl} \alpha_W^4 c \sim 10^{11} \mathrm{GeV} \ . \tag{22}$$

An important question concerns the accuracy of the numerical result presented in Fig. 2. Within the framework of the three-dimensional theory, there are essentially two sources of possible corrections. First, there might be higher order corrections to the masses calculated from the resummed action, eq. (9). The coefficient of the vector boson mass m_0 enters the prefactor exponentially so that corrections to this quantitiy are expected to alter the value of c significantly. Other changes, like small shifts in λ_{3R}/g_3^2 also due to mass corrections, should only have a minor impact in comparison. A quantitative estimate of the leading effect may be obtained by substituting $m_0 \to m_0(1+\gamma)$ in eq. (20), with $\gamma = \delta m_0/m_0$ varying between, say, -0.5 and 0.5. One finds that the prefactor varies accordingly between ~ 0.1 and ~ 0.0001, respectively, at a fixed temperature $T = 10T_b$.

Or, to put it the other way round, the result $c \sim \mathcal{O}(0.01)$ is stable as long as $\gamma \leq 0.2$. Hence, the order of magnitude of the coefficient c cannot be considered to be fixed until the dynamical vector boson mass is determined with sufficient accuracy. Second, there is the question of the applicability of the Langer-Affleck formula (2). If the saddle point expansion is to be valid, the one-loop contribution should be small compared to the saddle point contribution. In this calculation the fluctuations make up less than $\sim 22\%$ of the saddle point energy, for all values of T. This behaviour is improved for larger vector boson mass but worse for smaller vector boson mass, which also follows from the the effective loop expansion parameter, g_3^2/m.

Finally, the role of fermions and non-zero Matsubara modes needs to be discussed. The three-dimensional effective action (5) approximates the full four-dimensional one up to terms of the order $\sim m(T)/T$ [18], where $m(T)$ now is a generic temperature dependent mass $m \sim g^n T$ of any particle of the original theory. For fermions, scalars and the zero component of the gauge field $n = 1$, while for the spatial vector bosons $n = 2$ in the "symmetric" phase, so corrections are suppressed by powers of coupling constants. However, $g \sim 2/3$ is not very small and the corrections might be important numerically. Moreover, since these are corrections to the three-dimensional action which, up to a normlization, coincides with the sphaleron energy functional, their effect on the rate may be quite substantial. In Ref. [8] the sphaleron fluctuation determinants were calculated for the four-dimensional theory at finite temperature in the broken phase, including also a doublet of fermions. Indeed it was found that the fermion determinant enhances the sphaleron energy by about 30%, leading to an additional suppression of the rate. Since the scenario of the "symmetric" phase discussed in this paper resembles the broken phase in several respects, a similar effect may occur here too. An investigation of the full four-dimensional theory should be possible in principle, albeit technically complicated. It would require a separation of the zero and non-zero Matsubara modes which after the resummation of the zero mode sector could not be treated on the same footing anymore. Instead, a separate diagonalization of the corresponding fluctuation operators would be necessary.

In summary, the gap equation approach suggested in Ref. [15] was applied to the problem of B+L violation in the "symmetric" phase of the Standard Model and found to be a useful calculational method. It yields the expected asymptotic behaviour of the rate as well as the temperature where this behaviour sets in, and the numerical coefficient of the rate. The value of the coefficient turns out to be sensitive to higher order corrections to the dynamical vector boson mass. Given this uncertainty the numerical result obtained

at one-loop level may be regarded as compatible with previous results from lattice Monte-Carlo simulations [10]. In order to tighten the results obtained here the underlying picture of the "symmetric" phase needs to be confirmed by some other non-perturbative method. Once this is achieved, one can go on and attempt to generalize to the four-dimensional case including fermions.

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Figure captions

Fig.1 The resummed three-dimensional scalar coupling λ_{3R}/g_3^2 as a function of temperature for $\lambda/g^2 = 1/8$.

Fig.2 The coefficient c of the sphaleron rate (3) as a function of temperature. The solid line corresponds to $\lambda/g^2 = 1/8$ and the dashed line to $\lambda/g^2 = 1/64$.

Fig.1

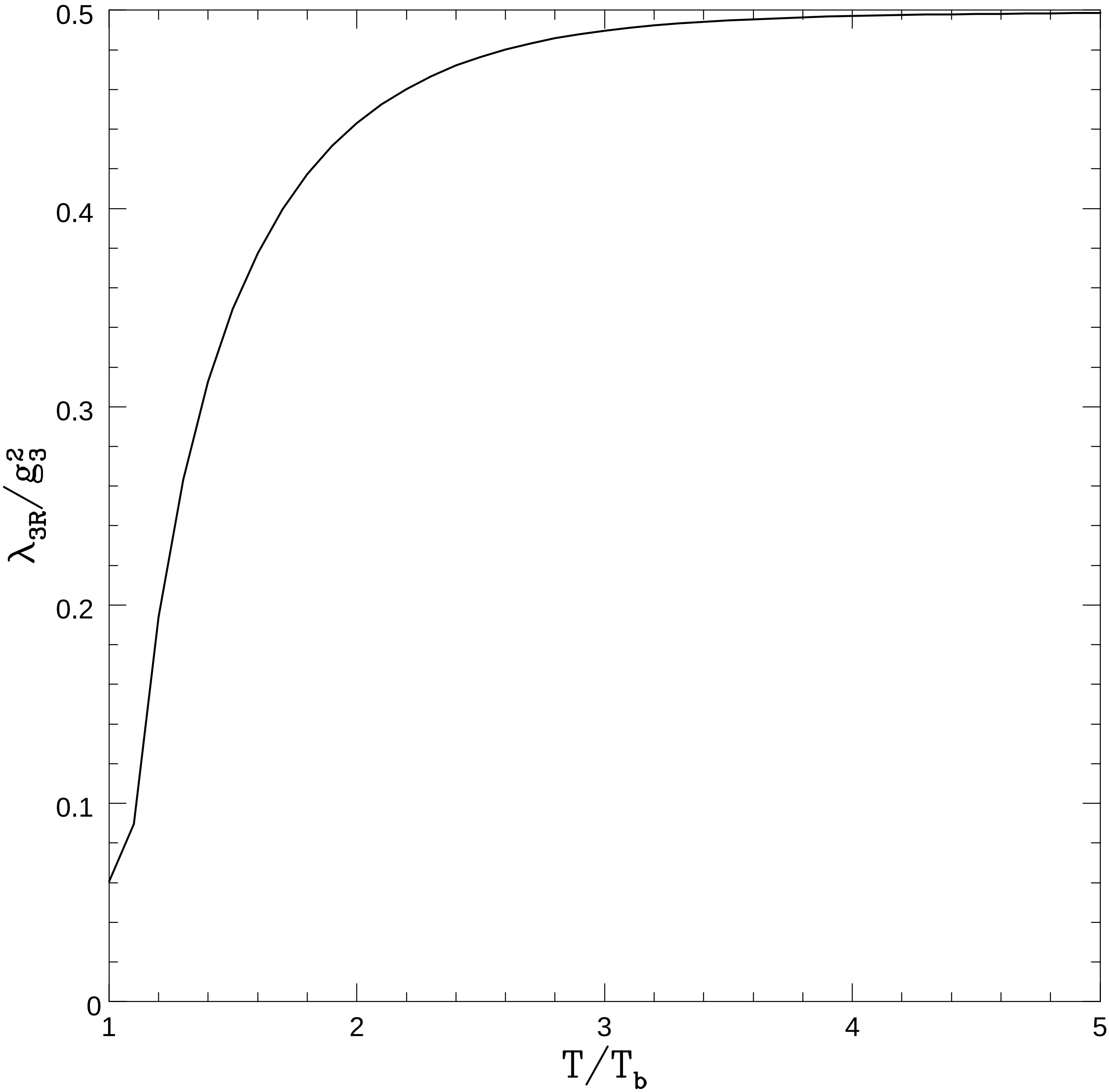

Fig.2